Name: ________________________ **MATH 158-03 F16** **Review for Exam 1**

Exam 1 will be similar to WebAssign, written homework, and examples done in class from the following sections.

12.1 Three-Dimensional Coordinate Systems

12.2 Vectors

12.3 The Dot Product

12.4 The Cross Product

12.5 Equations of Lines and Planes

12.6 Cylinders and Quadric Surfaces

13.1 Vector Functions and Space Curves

13.2 Derivatives and Integrals of Vector Functions

13.3 Arc Length and Curvature

13.4 Motion in Space: Velocity and Acceleration

Here is a brief list of topics but this list may not include all topics that may appear on the Exam.

12.1 Three-Dimensional Coordinate Systems

The **distance** from P_1 to P_2 is $|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

The equation of a **sphere** with center $C(h, k, l)$ and radius r is $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

12.2 Vectors

The **magnitude** of a vector is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Vector **sum** is $\mathbf{a} + c\mathbf{b} = \langle a_1 + cb_1, a_2 + cb_2, a_3 + cb_3\rangle$.

Properties of Vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ with scalars c and d.

$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$	$\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$	$\mathbf{a} + \mathbf{0} = \mathbf{a}$	$\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
$c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$	$(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$	$(cd)\mathbf{a} = c(d\mathbf{a})$	$1\mathbf{a} = \mathbf{a}$

The **unit vector** of $\mathbf{a}$ has length 1 and is in the same direction, is given by $\mathbf{u} = \frac{\mathbf{a}}{|\mathbf{a}|}$.

12.3 The Dot Product

The **dot product** is $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. Also, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$ where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. Two vectors are orthogonal if and only if their dot product is zero.

Properties of Dot Product

$\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$	$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$	$\mathbf{0} \cdot \mathbf{a} = 0$
$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$	$(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$	

Direction Angles and Cosines: $\frac{1}{|\mathbf{a}|}\mathbf{a} = \langle \cos\alpha, \cos\beta, \cos\gamma\rangle$

Vector Projections:

The scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$.

The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}$.

Work: $W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos\theta$

12.4 The Cross Product

The **cross product** is

$$\mathbf{a}\times\mathbf{b} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \langle a_2b_3 - a_3b_2,\ -(a_1b_3 - a_3b_1),\ a_1b_2 - a_2b_1\rangle.$$

The vector $\mathbf{a}\times\mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$. Two vectors are parallel if and only if their cross product is zero. If θ is the angle between, then $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$.

Properties of the Cross Product

$\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$	$(c\mathbf{a})\times\mathbf{b} = c(\mathbf{a}\times\mathbf{b}) = \mathbf{a}\times(c\mathbf{b})$
$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$	$(\mathbf{a}+\mathbf{b})\times\mathbf{c} = \mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}$

Torque is $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$ and so $|\boldsymbol{\tau}\,| = |\mathbf{r}\times\mathbf{F}| = |\mathbf{r}||\mathbf{F}|\sin\theta$.

12.5 Equations of Lines and Planes

A **line** with direction vector $\mathbf{v} = \langle a, b, c\rangle$ through P_0 is given by the following.

Vector equation: $\mathbf{r} = \mathbf{r_0} + t\mathbf{v}$

Parametric equations: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

Symmetric equations: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

Note that if given two points P_0 and P_1, then the direction vector $\mathbf{v} = \overrightarrow{P_0P_1} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$.

The **line segment** from $\mathbf{r}_0$ to $\mathbf{r}_1$ is $\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1$ for $0 \le t \le 1$.

A **plane** with normal vector $\mathbf{n} = \langle a, b, c\rangle$ that contains the point P_0 with position vector $\mathbf{r}_0$, is given by:

Vector equation: $\mathbf{n}\cdot(\mathbf{r} - \mathbf{r}_0) = 0$

Scalar equation: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ or $ax + by + cz + d = 0$

The **distance** between a point and a plane is $D = |\text{comp}_{\mathbf{n}}\mathbf{b}| = \frac{|\mathbf{n}\cdot\mathbf{b}|}{|\mathbf{n}|} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

12.6 Cylinders and Quadric Surfaces

Parabolic Cylinder	$y = ax^2$	Elliptic Cylinder	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Hyperbolic Cylinder	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
Elliptic Paraboloid	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Hyperbolic Parabaloid	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
Cone	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Hyperboloid of One Sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperboloid of Two Sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$		

13.1 Vector Functions and Space Curves

A vector function $\mathbf{r}(t) = \langle r_1(t), r_2(t), r_3(t)\rangle$ traces out a space curve $C = \{(r_1(t), r_2(t), r_3(t)) \,|\, t \in \mathbb{R}\}$.

Limit: $\lim\limits_{t\to a} \mathbf{r}(t) = \left\langle \lim\limits_{t\to a} r_1(t),\, \lim\limits_{t\to a} r_2(t),\, \lim\limits_{t\to a} r_3(t) \right\rangle$

$\mathbf{r}$ is **continuous** at a if $\lim\limits_{t\to a} \mathbf{r}(t) = \mathbf{r}(a)$.

13.2 Derivatives and Integrals of Vector Functions

The **derivative** is $\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t) = \lim\limits_{h\to 0} \frac{\mathbf{r}(t+h)-\mathbf{r}(h)}{h} = \langle r_1'(t),\, r_2'(t),\, r_3'(t)\rangle$.

The **unit tangent vector** is $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$.

Derivative Rules

$[\mathbf{u}(t) \pm \mathbf{v}(t)]' = \mathbf{u}'(t) \pm \mathbf{v}'(t)$

$[c\mathbf{u}(t)]' = c\mathbf{u}'(t)$

$[f(t)\mathbf{u}(t)]' = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$

$[\mathbf{u}(t) \cdot \mathbf{v}(t)]' = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$

$[\mathbf{u}(t) \times \mathbf{v}(t)]' = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$

$[\mathbf{u}(f(t))]' = f'(t)\mathbf{u}'(f(t))$

The **integral** is $\int_a^b \mathbf{r}(t)\,dt = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \langle r_1(t_i^*)\Delta t,\, r_2(t_i^*)\Delta t,\, r_3(t_i^*)\Delta t\rangle = \left\langle \int_a^b r_1(t)\,dt\,, \int_a^b r_2(t)\,dt\,, \int_a^b r_3(t)\,dt \right\rangle$.

13.3 Arc Length and Curvature

Arc length of a vector function is $L = \int_a^b |\mathbf{r}'(t)|\,dt$ and the arc length function is $s(t) = \int_a^t |\mathbf{r}'(u)|\,du$.

Curvature is defined as $\kappa = \left|\frac{d\mathbf{T}}{ds}\right|$ so $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ and $\kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$.

The **unit normal** is $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$ and the **unit binormal** is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$.

13.4 Motion in Space: Velocity and Acceleration

Velocity is $\mathbf{v}(t) = \mathbf{r}'(t)$ and **acceleration** is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

For the trajectory of a projectile motion, $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j}+t\mathbf{v}_0+\mathbf{D} = [(v_0\cos\alpha)t+D_1]\mathbf{i}+\left[-\frac{1}{2}gt^2 + (v_0\sin\alpha)t + D_2\right]\mathbf{j}$ where $v_0 = |\mathbf{v}_0|$, $\mathbf{D}$ is the initial position vector, and $g = |\mathbf{a}| \approx 9.8$ m/s^2 or 32 ft/s^2.

Tangential and Normal Components of Acceleration: $\mathbf{a} = v'\mathbf{T} + \kappa v^2\mathbf{N}$ where $v = |\mathbf{v}|$, $a_{\mathbf{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$ and $a_{\mathbf{N}} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$.